4th Edition
Revised & Updated

A Handbook on

Computer Science & IT

Contains well illustrated formulae & key theory concepts

for

GATE, PSUs

& OTHER COMPETITIVE EXAMS

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Director's Message


B. Singh (Ex. IES)

During the current age of international competition in Science and Technology, the Indian participation through skilled technical professionals have been challenging to the world. Constant efforts and desire to achieve top positions are still required.

I feel every candidate has ability to succeed but competitive environment and quality guidance is required to achieve high level goals. At MADE EASY, we help you to discover your hidden talent and success quotient to achieve your ultimate goals. In my opinion IAS, ESE, GATE & PSU's exams are tool to enter in to main stream of Nation serving. The real application of knowledge and talent starts, after you enter in to the working system. Here in MADE EASY you are also trained to become winner in your life and achieve job satisfaction.

MADE EASY aluminae have shared their winning stories of success and expressed their gratitude towards quality guidance of MADE EASY. Our students have not only secured All India First Ranks in ESE, GATE and PSU entrance examinations but also secured top positions in their career profiles. Now, I invite you to become aluminae of MADE EASY to explore and achieve ultimate goal of your life. I promise to provide you quality guidance with competitive environment which is far advanced and ahead than the reach of other institutions. You will get the guidance, support and inspiration that you need to reach the peak of your career.

I have true desire to serve Society and Nation by way of making easy path of the education for the people of India.

After a long experience of teaching in Computer Science & IT over the period of time MADE EASY team realised that there is a need of good *Handbook* which can provide the crux of Computer Science & IT in a concise form to the student to brush up the formulae and important concepts required for GATE and other competitive examinations. This *handbook* contains all the formulae and important theoretical aspects of Computer Science & IT. It provides much needed revision aid and study guidance before examinations.

B. Singh (Ex. IES)
CMD, MADE EASY Group

A Handbook on
Computer Science & IT

Contents

Contents

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***** Marked are not in GATE syllabus***

1

Discrete and Engineering Mathematics

I Mathematical Logic

Introduction

- **Proposition:** It is a declarative statement either TRUE or FALSE.
- **Compound Proposition:** It is a proposition formed using the logical operators (Negation ($\neg$), Conjunction ($\wedge$), Disjunction ($\vee$), etc.) with the existing propositions.
- **Logical Operators:**

  (*i*) Negation of p : $\neg p$ or $\bar{p}$ or $\sim p$

  (*ii*) Conjunction of p and q : $p \wedge q$

  (*iii*) Disjunction of p and q : $p \vee q$

  (*iv*) Implication/Conditional : $p \to q$ (if p, then q)

  (*v*) Bi-conditional : $p \leftrightarrow q$
- Precedence order of logical operators from high to low: $\neg, \wedge, \vee, \to, \leftrightarrow$
- $P \oplus R = PR' + P'R$, $P \leftrightarrow R = P'R' + PR$
- Number of distinct boolean expression with n variable = 2^{2^n}.
- **Normal form:** PCNF ($\vee$) = (POS = 0), PDFL ($\wedge$) = (SOP = 1)

  Total size = 2^n with n variable.

Note:

☑ Converse of $p \to q$ is : $q \to p$

☑ Inverse of $p \to q$ is : $\neg p \to \neg q$

☑ Contrapositive of $p \to q$ is : $\neg q \to \neg p$

Tautology

If compound proposition is always true then it is tautology.

Example: $p \vee \neg p$

Contradiction

If compound proposition is always false then it is contradiction.

Example: $p \wedge \neg p$

Contingency

Neither tautology nor contradiction.

Example: p

Logical Equivalence

$P \Leftrightarrow Q$ is tautology iff P and Q are logically equivalent.

Functionally Complete

If any formula can be written as an equivalent formula containing only the connectives in a set of operators, then such a set of operators is called as functionally complete.

Example:

$\{\uparrow\}$, $\{\downarrow\}$, $\{\neg, \vee\}$, $\{\neg, \wedge\}$, $\{\neg, \vee, \wedge\}$ are functionally complete (NAND).

Consistent

If $H_1 \wedge H_2 \wedge H_3 \wedge .... \wedge H_n$ is satisfiable then H_1, H_2, ... and H_n are consistent (Tautology, contingency but not contradiction).

Inconsistent

If $H_1 \wedge H_2 \wedge H_3 \wedge .... \wedge H_n$ is unsatisfiable then H_1, H_2, ... and H_n are inconsistent (only contradiction).

- **Valid:** Tautology, **Satisfiable:** Tautology + contingency, Invalid: contradiction + contingency, **Unsatisfiable:** Contradiction
- Sufficient ($\rightarrow$), necessary ($\leftarrow$), but = and, if = when = whenever, is = will = would = are = p unless q.
- $p \rightarrow q \equiv q$ unless $\neg p$ = "q is true unless p is false" either p is not true or q is true.
- p is necessary but not sufficient for $q = (q \rightarrow p) \wedge (p \rightarrow q)' = p \wedge q'$.

Equivalences

$$P \vee (P \wedge Q) \equiv P$$

$$P \rightarrow Q \equiv \neg P \vee Q \equiv \neg Q \rightarrow \neg P$$

$$P \wedge (P \vee Q) \equiv P$$

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$$

$$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

$$P \leftrightarrow Q \equiv \neg P \leftrightarrow \neg Q$$
$$P \to (Q \to R) \equiv (P \wedge Q) \to R$$
$$\neg(P \leftrightarrow Q) \equiv P \leftrightarrow (\sim Q) \equiv (\sim P) \leftrightarrow Q \equiv P \oplus Q$$
$$(P \to Q) \wedge (P \to R) \equiv P \to (Q \wedge R)$$
$$(P \to R) \wedge (Q \to R) \equiv (P \vee Q) \to R$$
$$(P \to Q) \vee (P \to R) \equiv P \to (Q \vee R)$$
$$(P \to R) \vee (Q \to R) \equiv (P \wedge Q) \to R$$
$$P \vee Q \equiv \neg P \to Q$$
$$P \wedge Q \equiv \neg(P \to \neg Q)$$
$$\neg(P \to Q) \equiv (P \wedge \neg Q)$$

Identity Laws: (i) $P \wedge T = P$, (ii) $P \vee F = P$

Domination Laws: (i) $P \vee T = T$, (ii) $P \wedge F = F$

Idempotent Laws: (i) $P \wedge P = P$, (ii) $P \vee P = P$

Commutative Laws:

(i) $P \vee Q = Q \vee P$, (ii) $P \wedge Q = Q \wedge P$

Associative Laws:

(i) $(P \vee Q) \vee R = P \vee (Q \vee R)$

(ii) $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$

Distributive Laws:

(i) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

(ii) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$

Demorgan's Laws:

(i) $\neg(P \wedge Q) = \neg P \vee \neg Q$

(ii) $\neg(P \vee Q) = \neg P \wedge \neg Q$

Absorption Laws:

(i) $P \vee (P \wedge Q) = P$, (ii) $P \wedge (P \vee Q) = P$

Negation Laws:

(i) $P \vee \neg P = T$, (ii) $P \wedge \neg P = F$

Double Negation Laws: $\neg(\neg P) = P$

Rules of Inference (Tautological Implications)

Simplification:

$$(P \wedge Q) \Rightarrow P$$
$$(P \wedge Q) \Rightarrow Q$$

Addition:

$$P \Rightarrow (P \vee Q)$$
$$Q \Rightarrow (P \vee Q)$$

Disjunctive Syllogism: $(\sim P, P \vee Q) \Rightarrow Q$

Modus Ponens: $(P, P \rightarrow Q) \Rightarrow Q$

Modus Tollens: $(\sim Q, P \rightarrow Q) \Rightarrow \sim P$

Hypothetical Syllogism: $(P \rightarrow Q, Q \rightarrow R) \Rightarrow (P \rightarrow R)$

Conjunctive Syllogism: $((P \vee Q), P) \Rightarrow \sim Q$

Dilemma: $(P \vee Q, P \rightarrow R, Q \rightarrow R) \Rightarrow R$

Constructive Dilemma: $(P \vee Q, P \rightarrow R, Q \rightarrow S) \Rightarrow R \vee S$

Destructive Dilemma: $(\sim R \vee \sim S, P \rightarrow R, Q \rightarrow S) \Rightarrow \sim P \vee \sim Q$

Other rules:

$$\sim P \Rightarrow (P \rightarrow Q)$$

$$Q \Rightarrow (P \rightarrow Q)$$

$$\sim(P \rightarrow Q) \Rightarrow P$$

$$\sim(P \rightarrow Q) \Rightarrow \sim Q$$

Exactly one = $\exists!$ or $\exists x\,[P(x) \wedge P(y) \Rightarrow y = x]$,

$$\forall x[\exists y\,(B(x, y) \wedge (B(x, z) \rightarrow y = z)$$

$p \Rightarrow q \Rightarrow r \equiv (p \wedge q) \rightarrow r = q \Rightarrow (p \Rightarrow r)$

Principle Conjunctive Normal Form (PCNF)

Product of sums (max term)

PCNF: $[P(x_1) \vee P(x_2)] \wedge [P(x_3) \vee P(x_4)]$

Principle Disjunctive Normal Form (PDNF)

Sums of products (min term)

PDNF: $[P(x_1) \wedge P(x_2)] \vee [P(x_3) \wedge P(x_4)]$

Number of non equivalent propositional functions with n-propositional variables are = 2^{2^n}.

- $\forall x\,(\alpha \rightarrow \beta) \Rightarrow (\forall x\,\alpha \Rightarrow \forall x\,\beta)$ true only with properties always use and but not $\rightarrow$.

Predicate Logic

Quantifiers

- **Universal ($\forall$) :** "for all" or "for every"
- **Existential ($\exists$) :** "there exist"

Predicates

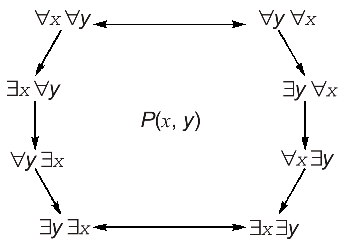
- $P(x)$: Propositional statement with one variable.
- $Q(x, y)$: Propositional statement with two variables.

Note:

☑ $\neg\exists x\, P(x) = \forall x\, \neg P(x)$

☑ $\neg\forall x\, P(x) = \exists x\, \neg P(x)$

Logical Equivalences



1. $\forall x\, [P(x) \wedge Q(x)] \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$
2. $\exists x [P(x) \vee Q(x)] \equiv \exists x\, P(x) \vee \exists x\, Q(x)$
3. $\forall x\, (P(x) \vee Q) \equiv \forall x\, P(x) \vee Q$
4. $\forall x\, (P(x) \wedge Q) \equiv \forall x\, P(x) \wedge Q$
5. $\exists x\, (P(x) \vee Q) \equiv \exists x\, P(x) \vee Q$
6. $\exists x\, (P(x) \wedge Q) \equiv \exists x\, P(x) \wedge Q$
7. $\forall x\, P(x) \wedge \exists y Q(y) \equiv \forall x \exists y [P(x) \wedge Q(y)]$
8. $\forall x\, P(x) \vee \exists y Q(y) \equiv \forall x \exists y [P(x) \vee Q(y)]$

II Combinatorics

Permutations (Ordered Selection/Arrangement)

- The number of permutations of n-objects:
  1. Taken 'r' at a time = ${}^nP_r = P(n, r) = (n)_r$ (arrangement).
  2. Taken 'r' at a time = n^r (with repetition) (arrangement).
  3. Taken all at a time = $n!$

4. Taken not more than 'r' = $\dfrac{(n^{r+1}-1)}{n-1}$ (with repetition).

5. Taken 'r' (atleast one repeated) = All – none = $n^r - {}^nP_r$

6. Taken all at a time, in which 'r' of them are alike (identical) = $\dfrac{n!}{r!}$.

7. Taken all at a time, in which n_1 are alike, n_2 are alike, ..., n_r are alike

$$= \frac{n!}{n_1!\, n_2! \ldots n_r!}$$

8. Taken 'r' at a time, in which m-particular objects are:
   (a) Never included = ${}^{(n-m)}P_r$.
   (b) Always included ($n \geq m$ and $r \geq m$) = ${}^{(n-m)}P_{(r-m)} \cdot {}^rP_m$.

9. ${}^nP_r = P(n-1, r) + r \cdot P(n-1, r-1)$

10. ${}^nP_r = r! \cdot {}^nC_r = n \times {}^{(n-1)}P_{(r-1)}$

11. n types of objects (each of infinity in number)

$$\underbrace{n_1 + n_2 + n_3 + n_4 + \ldots.. + n_n}_{n} = r = {}^{n-1+r}C_r$$

12. m boys and n girls are arrange in straight line = $(m + n - 1)!$

- **The Number of Circular Permutations of *n* object:**

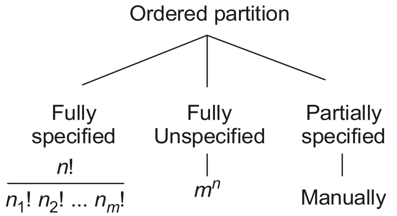
1. Taken all at a time = $(n-1)!$; (If not directed)

$= \dfrac{(n-1)!}{2}$; (If direction (clockwise/anti-clockwise) is given)

2. Taken 'r' at a time = $\dfrac{{}^nP_r}{r}$; (If not directed), and $\dfrac{1}{2} \cdot \dfrac{{}^nP_r}{r}$; (if directed)

3. Necklace with m identical beads, n identical beads in circular number of way = $\dfrac{(m+n-1)}{m! \times n! \times 2!}$.

Distinct → Distinct → Ordered partition	Distinct → Indistinct → Unordered	Indistinct → Distinct → Ball in box
$\dfrac{n!}{n_1!\, n_2! \ldots n_n!}$	$\dfrac{n!}{(G!)^P}$	$\dfrac{n!}{(G!)^P \times P!}$



Derangements

- Number of derangements for n-objects = $D_n = n!\sum_{i=0}^{n}\frac{(-1)^i}{i!}$

$$\left(D_n \cong \frac{n!}{e} \text{ for large value of } n\right) = \left[\frac{n!}{2!}-\frac{n!}{3!}+\frac{n!}{4!}.....\frac{n!}{n!}\right]$$

$$\left(\text{The prob. of a given permutation being a derangement} = \frac{D_n}{n!} = \sum_{i=0}^{n}\frac{(-1)^i}{i!} \cong \frac{1}{e} \text{ for large } n\right)$$

- **Binomial theorem:**

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1}b + \ldots + {}^nC_n\, a^0 b^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

- **Multinomial coefficients:**

$$\binom{n}{n_1,\ n_2,\ldots,n_r} = \frac{n!}{n_1!\,n_2!\ldots.n_r!}$$

Note: ....

☑ Number of ways to distribute 'r' apples to n-persons = ${}^{(n+r-1)}C_r$.

☑ The number of permutations of (n_1+n_2) objects, taken all at a time, in which n_1 objects are alike and n_2 objects are alike = $\frac{(n_1+n_2)!}{n_1!\,n_2!}$.

....

Combinations (Unordered Selection)

$$^nC_r = C(n,r) = \frac{n!}{(n-r)!\,r!}$$

$$C(n,r) = C(n-1,\ r) + C(n-1, r-1)$$

- **The number of combinations of n-objects:**

(*i*) Taken (selecting) 'r' at a time = ^nC_r

(*ii*) Taken (selecting zero or more at a time) = 2^n

(*iii*) Taken one or more at a time = $2^n - 1$

(*iv*) Where the objects are divided into p-objects of one type, q-objects of second type and so on; A non-empty selection from this can be made in $[(p+1)(q+1)......] -1$ ways.

Digital Logic

2

I Logic Functions

Logic Gates

- OR, AND, NOT are Basic Gates
- NAND, NOR are Universal Gates
- EX-OR, EX-NOR are Arithmetic Gates

NOT Gate

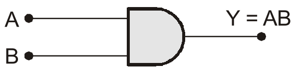
- Also referred to as “Inversion” or “Complementation”.

  Symbol and Truth table:

Inputs A	Output Y = $\overline{A}$
0	1
1	0

AND Gate

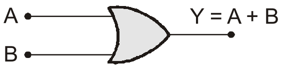
Symbol and Truth table:



Inputs		Output
A	B	Y = AB
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

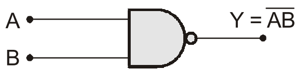
Symbol and Truth table:



Inputs		Output
A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

NAND Gate

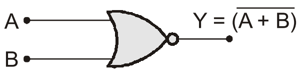
Symbol and Truth table:



Inputs		Output
A	B	Y = $\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate

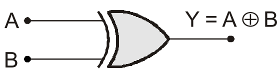
Symbol and Truth table:



Inputs		Output
A	B	Y = $\overline{(A + B)}$
0	0	1
0	1	0
1	0	0
1	1	0

EX-OR Gate

- It is also called “stair case switch”.
- Symbol and Truth table:



Inputs		Output
A	B	Y = A ⊕ B
0	0	0
0	1	1
1	0	1
1	1	0

- Boolean function of 2-input EXOR operation:

$$Y = A \oplus B = \bar{A}B + A\bar{B}$$

(*i*) It acts like as an “odd number of 1’s detector in the input”.

(*ii*) It is mostly used in “parity generation and detection”.

(*iii*) When both the inputs are same, then output becomes LOW or Logic ‘0’.

(*iv*) When both the inputs are different, then output becomes HIGH or Logic ‘1’.

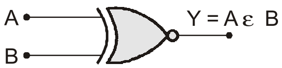
(*v*) $A \oplus A = 0, \quad A \oplus 0 = A$

$A \oplus \bar{A} = 1, \quad A \oplus 1 = \bar{A}$

- $A \oplus A \oplus A \oplus$ ... upto n terms = 0, when n = even
- $A \oplus A \oplus A \oplus$ ... upto n terms = A, when n = odd

EX-NOR Gate

- It acts like as an "even number of 1's detector".
- Symbol and Truth table:



Inputs		Output
A	B	Y = A ε B
0	0	1
0	1	0
1	0	0
1	1	1

- Boolean function of 2-input EX-NOR operation:

$$Y = A \odot B = \overline{A \oplus B} = (\overline{\bar{A}B + A\bar{B}}) = AB + \bar{A}\bar{B}$$

(*i*) When both the inputs are same, then output becomes HIGH or Logic '1'.

(*ii*) When both the inputs are different, then output becomes LOW or Logic '0'.

(*iii*) $A \odot A = 1, \quad A \odot 1 = A$

$A \odot \bar{A} = 0, \quad A \odot 0 = \bar{A}$

- $A \odot A \odot A \odot$ ... upto n terms = 1, when n is even
- $A \odot A \odot A \odot$ ... upto n terms = A, when n is odd

Note: ....

☑ $\bar{A} \oplus B = A \oplus \bar{B} = A \odot B$

☑ $\bar{A} \odot B = A \odot \bar{B} = A \oplus B$

☑ For odd number of inputs EX-OR and EX-NOR are same and for even number of variables they are complements to each other.

i.e. $A \oplus B \oplus C = A \odot B \odot C$

as $A \oplus B \oplus C = \overline{A \oplus B}\, C + (A \oplus B)\bar{C}$

$= (A \odot B)\, C + (A \oplus B)\bar{C}$

$= (A \odot B)C + \overline{A \odot B}\, \bar{C}$

$= A \odot B \odot C$

☑ $\overline{A \oplus B \oplus C} = A \odot B \oplus C = A \oplus B \odot C$

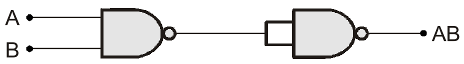
☑ $\overline{A \odot B \odot C} = A \oplus B \odot C = A \odot B \oplus C$

☑ $A \oplus B \oplus AB = A + B$

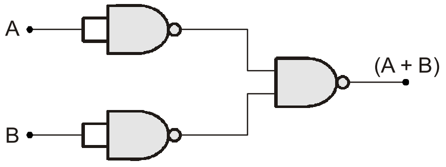
☑ EX-OR and EX-NOR are also called arithmetic gates as they are used in addition, subtraction, comparator circuits.

Implementation of Gates using NAND

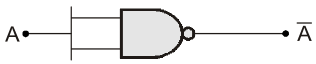
- AND gate:



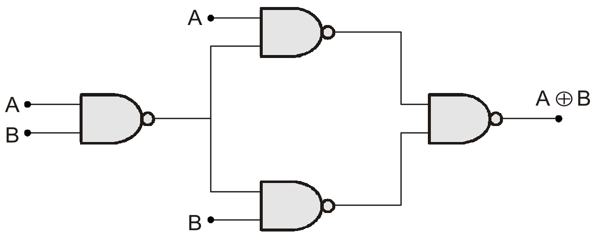
- OR gate:



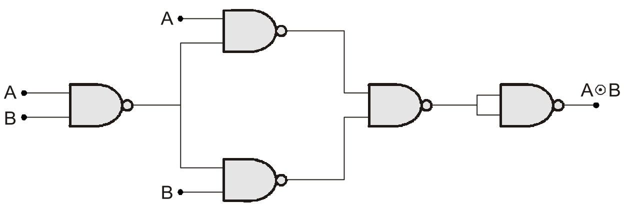
- NOT gate:



- EX-OR gate:

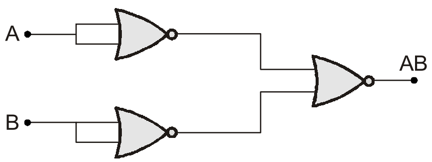


- EX-NOR Gate:

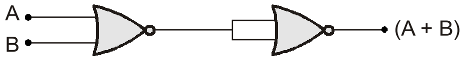


Implementation of Gates using NOR

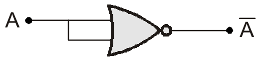
- AND gate:



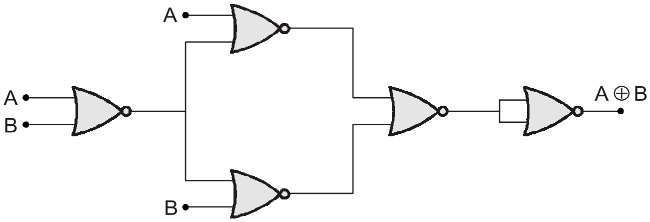
- OR gate:



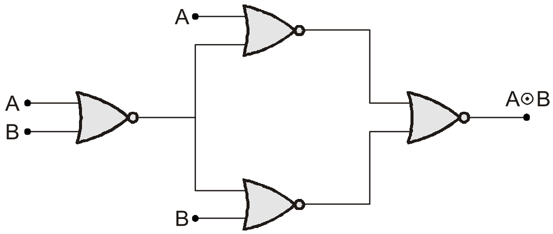
- NOT gate:



- EX-OR gate:



- EX-NOR gate:



Note:

☑ Number of NAND and NOR gates needed to implement other logic gates is shown in the following table.

Logic gate	No. of NAND gates	No. of NOR gates
NOT	1	1
AND	2	3
OR	3	2
EX OR	4	5
EX NOR	5	4

☑ Alternative Symbols of Gates

1. Bubbled – OR gate ≡ NAND gate
2. Bubbled – NAND gate ≡ OR gate
3. Bubble – NOR gate ≡ AND gate
4. Bubbled – AND gate ≡ NOR gate